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International Journal of Thermal Sciences 44 (2005) 303–310
International Journal of Thermal Sciences
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Exergetic efficiency optimization for an irreversible Brayton refrigeration cycle

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Received 10 May 2004; received in revised form 8 September 2004; accepted 8 September 2004

Available online 18 November 2004

Abstract

Exergetic efficiency optimization that combines exergy concept and finite-time thermodynamic theory has been carried out for an irreversible Brayton refrigeration cycle. Multi-irreversibilities considered in the system include finite rate heat transfer, internal dissipation of the working fluid and heat leak between heat reservoirs. Exergetic efficiency defined as the ratio of rate of exergy output to rate of exergy input of the system is considered as the objective index. The goal of exergetic efficiency optimization is to maximize this index. The maximum value of the exergetic efficiency can be determined analytically. The results are compared with those obtained from the traditional coefficient of performance. The influences of heat leak between heat reservoirs and temperature ratio of two reservoirs on the exergetic efficiency are investigated by numerical calculations. The allocation of a fixed total thermal conductance between the two heat exchangers is also discussed. The results show that the method of exergetic efficiency optimization is an important and effective criterion for the evaluation of an irreversible Brayton refrigeration cycle.
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Keywords: Exergetic efficiency; Optimization; Irreversible; Brayton; Refrigeration cycle

1. Introduction

Finite time thermodynamics is more practical than classical thermodynamics for evaluating the power output and thermal efficiency of a thermodynamic cycle [1]. Many important works about Brayton power or refrigeration cycles applying finite-time thermodynamic theory have been published in recent years. De Vos [2] investigated the efficiency of some heat engines at maximum power conditions. Bejan [3] built the theory of heat transfer-irreversible refrigeration plants. Wu [4] defined the endoreversible heat engine and optimized the power output of an endoreversible Brayton heat engine. Sahin et al. [5] analyzed the maximum power density of an irreversible Joule–Brayton engine. Wu et al. [6] analyzed and optimized the cooling load of the endoreversible simple Brayton refrigeration cycles coupled to constant- and variable-temperature heat reservoirs, and compared the performance with that of endoreversible Carnot refrigeration cycles coupled to constant- and variable-temperature heat reservoirs. Chen et al. [7] analyzed the performance of a regenerative closed Brayton cycle. The analysis considered all the irreversibilities associated with finite-time heat transfer processes. Chen et al. [8] analyzed the cooling load and *COP* performance of endoreversible regenerated Brayton refrigeration cycles coupled to constant- and variable-temperature heat reservoirs. Cheng and Chen [9] determined the maximum power output and the corresponding thermal efficiency for an irreversible closed-cycle Brayton heat engine. Later, Cheng and Chen [10] calculated the maximum thermal efficiency and the corresponding power output for the same system. Chen et al. analyzed the cooling load and *COP* performance of irreversible simple [11] and regenerated [12] Brayton refrigeration cycles coupled to constant- and variable-temperature heat reservoirs. Sahin et al. [13] analyzed a comparative performance of irreversible regenerative reheating Joule–Brayton engines

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doi:10.1016/j.ijthermalsci.2004.09.003

Nomenclature

A	heat transfer area	m^2
C_1–C_{16}	coefficients	
$\dot{C}_I$	heat transfer rate between reservoirs	$W \cdot K^{-1}$
$\dot{C}_w$	heat capacitance rate of working fluid	$W \cdot K^{-1}$
COP	coefficient of performance	
$\dot{E}$	rate of exergy	W
N	number of transfer units of heat exchangers	
$\dot{Q}$	rate of heat transfer	W
$\dot{Q}_I$	rate of heat leak	W
T	temperature	K
U	heat transfer coefficient of heat exchanger	$W \cdot m^{-2} \cdot K^{-1}$
$\dot{W}_{in,net}$	net power input	W
y	isentropic temperature ratio	
z	allocation factor	

Greek symbols

ε	effectiveness of heat exchanger
η	efficiency

Subscripts

0	typical environment
c	compressor
d	destruction
e	expansion
ex	exergetic
H	hot side
HC	process at hot side
in	input
L	cold side
LC	process at cold side
max/opt	maximum/optimum condition
out	output

under maximum power density and maximum power conditions. The performance of real regenerated air heat pumps was analyzed by Chen et al. [14]. Cheng and Chen [15] determined the maximum ecological function, its corresponding thermal efficiency and power output of an irreversible Brayton heat engine. The ecological function of a heat engine was defined as the power output minus the loss power. Chen et al. [16] analyzed the performance of a closed regenerated Brayton heat pump with internal irreversibilities via methods of entropy generation minimization. An exergy analysis based on an ecological optimization criterion was carried out for an irreversible Brayton engine with an external heat source by Huang et al. [1]. Kodal et al. [17] investigated the effects of internal irreversibility and heat leakage on the finite time thermoeconomic performance of refrigerators and heat pumps. Chen et al. [18] optimized the power density for an irreversible regenerated closed Brayton cycle. Chen et al. [19] analyzed and optimized the power density of an irreversible regenerated closed Brayton cycle coupled to variable-temperature heat reservoirs. Chen et al. [20] optimized the power density of an irreversible closed Brayton cycle coupled to constant-temperature heat reservoirs in the viewpoint of entropy generation minimization. Luo et al. [21] optimized cooling load and *COP* performance of irreversible simple Brayton refrigeration cycle coupled to constant-temperature heat reservoirs. Zhou et al. analyzed and optimized cooling load density of the endoreversible simple Brayton refrigeration cycles coupled to constant- [22] and variable- [23] temperature heat reservoirs, of the irreversible simple Brayton refrigeration cycle coupled to constant- [24] and variable- [25] temperature heat reservoirs, and of irreversible regenerated Brayton refrigeration cycles coupled to constant- [26] and variable- [27] temperature heat reservoirs. The cooling load density was defined as the ratio of cooling load to the maximum specific volume in the cycle.

In recent years, the research combining exergy concept and finite time thermodynamics is becoming increasingly important. Yan and Chen [28] optimized the rate of exergy output for an endoreversible Carnot refrigerator. Sahin et al. [29] also determined the optimum values of design parameters of the cogeneration cycle at maximum exergy output. Yilmaz [30] investigated the effects of design parameters on the exergetic performance for cogeneration systems with external irreversibilities. They carried out the optimum analysis on the performance of exergy, incorporating time or rate constraints in the conditions defining the system. However, the optimization of an irreversible Brayton refrigeration cycle based on the performance of exergetic efficiency has not been investigated.

In this paper, exergetic efficiency optimization for an irreversible Brayton refrigeration cycle is reported. The purpose of this paper is to maximize the exergetic efficiency of the refrigeration system. The multi-irreversibilities considered are finite rate heat transfer, internal dissipation of the working fluid and heat leak between heat reservoirs. The exergetic efficiency optimization performed in this paper helps to better understand the performance of the irreversible Brayton refrigeration cycle.

2. Theoretical model

A steady-flow irreversible Brayton refrigeration cycle coupled to two regions at a temperature T_L and another higher temperature T_H is shown in Fig. 1. The refrigeration cycle consists of two isobaric processes (processes 2-3 and 4-1) and two non-isentropic processes (the compres-

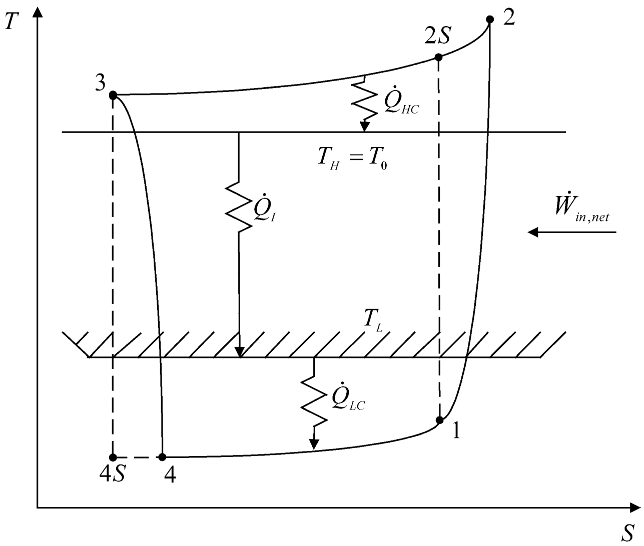

Fig. 1. The irreversible Brayton refrigeration cycle coupled to two reservoirs.

sion process 1-2 and the expansion process 3-4). The cycle 1-2-3-4-1 is irreversible due to the internal dissipation of the working fluid, the finite rate heat transfer, and the heat leak between the heat reservoirs, while 1-2S represents the corresponding isentropic compression and 3-4S represents the corresponding isentropic expansion during a reversible Brayton refrigeration cycle.

The infinite reservoir at temperature T_H is set to be the typical environment for simplicity. It gives

$$T_H = T_0 \tag{1}$$

where T_0 is the temperature of the typical environment. Moreover, the influences of kinetic energy and potential energy are assumed to be negligible.

The purpose of employing a refrigeration system is to absorb heat from a cold space. Thus, it increases the exergy output while heat is absorbed from the cold space to the refrigeration cycle. The rate of exergy output of the system is given as

$$\dot{E}_{out} = \dot{Q}_L\left(\frac{T_0}{T_L} - 1\right) - \dot{Q}_H\left(\frac{T_0}{T_H} - 1\right) \tag{2}$$

$$= \dot{Q}_L\left(\frac{T_H}{T_L} - 1\right) \tag{3}$$

It is also noted that the values of exergy output rates are different for equivalent heat transfer rates at various boundary temperatures.

When considering the interaction between the refrigeration cycle and its surrounding, the rate of exergy destruction is given as

$$\dot{E}_d = T_0\left(\frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H}\right) \tag{4}$$

where $\dot{Q}_L/T_L - \dot{Q}_H/T_H$ also represents the rate of entropy generation of the refrigeration system. According to the first law of thermodynamics, the net power input of the refrigeration cycle is

$$\dot{W}_{in,net} = \dot{Q}_H - \dot{Q}_L \tag{5}$$

which equals the rate of exergy input. The rate of exergy input is given as

$$\dot{E}_{in} = \dot{W}_{in,net} = \dot{Q}_H - \dot{Q}_L \tag{6}$$

To improve the performance of the refrigeration system, the exergetic efficiency η_{ex} defined as the ratio of rate of exergy output $\dot{E}_{out}$ to rate of exergy input $\dot{E}_{in}$ is taken as the objective index to be maximized. When Eqs. (3) and (6) are combined, the exergetic efficiency of the irreversible Brayton refrigeration cycle is obtained as

$$\eta_{ex} = \frac{\dot{Q}_L(T_H/T_L - 1)}{\dot{Q}_H - \dot{Q}_L} \tag{7}$$

To further optimize the exergetic efficiency, it is necessary to study the type of heat transfer occurring in the system. It is often assumed that heat transfer obeys a linear law. Employing the log mean temperature differences in the hot-side and cold-side heat exchangers, rates of heat transfer during the cycle can be given by the following

$$\dot{Q}_{HC} = U_H A_H \frac{(T_2 - T_H) - (T_3 - T_H)}{\ln[(T_2 - T_H)/(T_3 - T_H)]} = \dot{C}_w(T_2 - T_3) \tag{8}$$

$$\dot{Q}_{LC} = U_L A_L \frac{(T_L - T_4) - (T_L - T_1)}{\ln[(T_L - T_4)/(T_L - T_1)]} = \dot{C}_w(T_1 - T_4) \tag{9}$$

In the above, U_H and U_L are respectively the overall heat transfer coefficients of the hot-side and the cold-side heat exchangers, A_H and A_L are respectively the areas of the hot-side and the cold-side heat exchangers, while $\dot{C}_w$ is the thermal capacitance rate of the working fluid. The heat exchanger thermal conductance is defined as the heat transfer coefficient-area product UA. Moreover, the heat leak rate between the two heat reservoirs was first provided by Bejan [31]. It is given as

$$\dot{Q}_I = \dot{C}_I(T_H - T_L) \tag{10}$$

where $\dot{C}_I$ is the heat transfer rate between the two reservoirs per unit temperature difference.

From Eqs. (8) and (9), the numbers of transfer units of hot-side and cold-side heat exchangers are

$$N_H = \frac{U_H A_H}{\dot{C}_w} = \ln\frac{T_2 - T_H}{T_3 - T_H} \tag{11}$$

and

$$N_L = \frac{U_L A_L}{\dot{C}_w} = \ln\frac{T_L - T_4}{T_L - T_1} \tag{12}$$

The effectivenesses of hot-side and cold-side heat exchangers for counterflow heat exchangers are defined as [32]

$$\varepsilon_H = 1 - \exp(-N_H) \tag{13}$$

$$\varepsilon_L = 1 - \exp(-N_L) \tag{14}$$

Combining Eqs. (8), (9), (11)–(14) yields

$$T_3 = \varepsilon_H T_H + (1-\varepsilon_H)T_2 \tag{15}$$

$$T_1 = \varepsilon_L T_L + (1-\varepsilon_L)T_4 \tag{16}$$

and

$$\dot{Q}_{HC} = \dot{C}_w(T_2 - T_3) = \dot{C}_w \varepsilon_H (T_2 - T_H) \tag{17}$$

$$\dot{Q}_{LC} = \dot{C}_w(T_1 - T_4) = \dot{C}_w \varepsilon_L (T_L - T_4) \tag{18}$$

The heat transfer rates yielded by the hot reservoir and the cold reservoir are

$$\dot{Q}_H = \dot{Q}_{HC} - \dot{Q}_I \tag{19}$$

$$\dot{Q}_L = \dot{Q}_{LC} - \dot{Q}_I \tag{20}$$

where $\dot{Q}_L$ is also the cooling rate of the refrigeration cycle.

Moreover, the coefficient of performance (*COP*) of the refrigeration cycle can be given as

$$COP = \frac{\dot{Q}_L}{\dot{W}_{in,net}} = \frac{\dot{Q}_L}{\dot{Q}_H - \dot{Q}_L} = \frac{\dot{Q}_{LC} - \dot{Q}_I}{\dot{Q}_{HC} - \dot{Q}_{LC}} \tag{21}$$

Combining Eqs. (7), (10), (17)–(20) yields

$$\eta_{ex} = \frac{[\dot{C}_w \varepsilon_L (T_L - T_4) - \dot{C}_I (T_H - T_L)](T_H/T_L - 1)}{\dot{C}_w \varepsilon_H (T_2 - T_H) - \dot{C}_w \varepsilon_L (T_L - T_4)} \tag{22}$$

3. Optimization

For an irreversible Brayton refrigeration cycle, the isentropic compressor efficiency and isentropic expansion efficiency are introduced and defined as

$$\eta_c = \frac{T_{2S} - T_1}{T_2 - T_1} \tag{23}$$

$$\eta_e = \frac{T_3 - T_4}{T_3 - T_{4S}} \tag{24}$$

Applying the second law of thermodynamics to cycle 1-2S-3-4S-1 gives

$$y = \frac{T_{2S}}{T_1} = \frac{T_3}{T_{4S}} > \frac{T_H}{T_L} \tag{25}$$

where y is the isentropic temperature ratio of the Brayton refrigeration cycle.

Combining Eqs. (16), (24) and (25) gives

$$T_{4S} = T_4/\eta_e + (1 - 1/\eta_e)T_3 \tag{26}$$

$$T_4 = (1 - \eta_e + \eta_e/y)T_3 \tag{27}$$

$$T_1 = \varepsilon_L T_L + (1-\varepsilon_L)(1 - \eta_e + \eta_e/y)T_3 \tag{28}$$

Combining Eqs. (15) and (23) gives

$$T_{2S} = \eta_c T_2 + (1-\eta_c)T_1 \tag{29}$$

$$T_2 = T_3/(1-\varepsilon_H) - \varepsilon_H T_H/(1-\varepsilon_H) \tag{30}$$

Substitution of Eqs. (28), (29) into Eq. (25) and combining Eq. (30) yield

$$T_3 = \frac{C_4 y^2 + C_5 y}{C_1 y^2 + C_2 y + C_3} \tag{31}$$

where

$$C_1 = (1-\varepsilon_L)(1-\eta_e)$$

$$C_2 = (1-\varepsilon_L)\eta_e - \eta_c/(1-\varepsilon_H) - (1-\varepsilon_L)(1-\eta_c)(1-\eta_e)$$

$$C_3 = -\eta_e(1-\varepsilon_L)(1-\eta_c)$$

$$C_4 = -\varepsilon_L T_L$$

$$C_5 = -\varepsilon_H \eta_c T_H/(1-\varepsilon_H) + \varepsilon_L(1-\eta_c)T_L$$

To facilitate optimizing the exergetic efficiency by regarding the isentropic temperature ratio y as a parameter, Eq. (22) is rearranged as

$$\eta_{ex} = \frac{C_9 T_4 + C_{10}}{C_6 T_2 + C_7 T_4 + C_8} \tag{32}$$

where

$$C_6 = \dot{C}_w \varepsilon_H$$

$$C_7 = \dot{C}_w \varepsilon_L$$

$$C_8 = -\dot{C}_w(\varepsilon_H T_H + \varepsilon_L T_L)$$

$$C_9 = -\dot{C}_w \varepsilon_L (T_H/T_L - 1)$$

$$C_{10} = \left[\dot{C}_w \varepsilon_L T_L - \dot{C}_I (T_H - T_L)\right](T_H/T_L - 1)$$

Combining Eqs. (27), (30)–(32) gives

$$\eta_{ex} = \frac{C_{14} y^2 + C_{15} y + C_{16}}{C_{11} y^2 + C_{12} y + C_{13}} \tag{33}$$

where

$$C_{11} = C_4 C_7 (1-\eta_e) + C_4 C_6/(1-\varepsilon_H) + C_1\left[C_8 - C_6 \varepsilon_H T_H/(1-\varepsilon_H)\right]$$

$$C_{12} = C_4 C_7 \eta_e + C_5 C_7 (1-\eta_e) + C_5 C_6/(1-\varepsilon_H) + C_2\left[C_8 - C_6 \varepsilon_H T_H/(1-\varepsilon_H)\right]$$

$$C_{13} = C_5 C_7 \eta_e + C_3\left[C_8 - C_6 \varepsilon_H T_H/(1-\varepsilon_H)\right]$$

$$C_{14} = C_1 C_{10} + C_4 C_9 (1-\eta_e)$$

$$C_{15} = C_2 C_{10} + C_4 C_9 \eta_e + C_5 C_9 (1-\eta_e)$$

$$C_{16} = C_3 C_{10} + C_5 C_9 \eta_e$$

Maximizing η_{ex} by taking $\partial \eta_{ex}/\partial y = 0$ yields

$$(C_{12}C_{14} - C_{11}C_{15})y^2 + 2(C_{13}C_{14} - C_{11}C_{16})y + (C_{13}C_{15} - C_{12}C_{16}) = 0 \tag{34}$$

By solving Eq. (34), the optimum isentropic temperature ratio y_{opt} of the irreversible Brayton refrigeration cycle is obtained as

$$y_{opt} = \left[(C_{11}C_{16} - C_{13}C_{14}) - \left[(C_{11}C_{16} - C_{13}C_{14})^2 - (C_{12}C_{14} - C_{11}C_{15})(C_{13}C_{15} - C_{12}C_{16})\right]^{0.5}\right] \times [C_{12}C_{14} - C_{11}C_{15}]^{-1} \tag{35}$$

The optimum exergetic efficiency $\eta_{ex,opt}$ is then obtained by substituting Eq. (35) into Eq. (33) as

$$\eta_{ex,opt} = \frac{C_{14} y_{opt}^2 + C_{15} y_{opt} + C_{16}}{C_{11} y_{opt}^2 + C_{12} y_{opt} + C_{13}} \tag{36}$$

Moreover, substituting Eq. (35) into Eq. (31) yields the optimum cycle temperature $T_{3,\text{opt}}$. Then, other optimum cycle temperatures $T_{4,\text{opt}}$, $T_{1,\text{opt}}$, and $T_{2,\text{opt}}$ can be calculated by Eqs. (27), (28), and (30). The corresponding values, such as optimum net power input $\dot{W}_{\text{in,net,opt}}$, optimum cooling rate $\dot{Q}_{\text{L,opt}}$, and optimum coefficient of performance COP_{opt} can also be obtained.

In addition, the allocation of a fixed total thermal conductance between the two heat exchangers is considered. Dividing the total thermal conductance by $\dot{C}_w$ is set to be a fixed value N, as in the following

$$N = \frac{U_{\text{H}}A_{\text{H}} + U_{\text{L}}A_{\text{L}}}{\dot{C}_w} = N_{\text{H}} + N_{\text{L}} \tag{37}$$

Let N_{H} and N_{L} be allocated by allocation factor z, and they satisfy

$$N_{\text{H}} = zN \tag{38}$$

$$N_{\text{L}} = (1-z)N \tag{39}$$

Substituting Eqs. (38) and (39) into Eqs. (13) and (14) gives

$$\varepsilon_{\text{H}} = 1 - \exp(-zN) \tag{40}$$

$$\varepsilon_{\text{L}} = 1 - \exp\big[(z-1)N\big] \tag{41}$$

Eqs. (40) and (41) can be utilized to analyze the allocation of a fixed total thermal conductance between two heat exchangers numerically.

To see the results of exergetic efficiency optimization for an irreversible Brayton refrigeration cycle, some numerical examples are presented and discussed. In the following calculation, a temperature condition of $T_{\text{H}} = T_0 = 298.15$ K is assumed.

The exergetic efficiency η_{ex} of an irreversible Brayton refrigeration cycle is shown as a function of the isentropic temperature ratio y in Fig. 2. It is clearly seen that the maximum value of the exergetic efficiency occurs when the isentropic temperature ratio is slightly greater than the temperature ratio of two reservoirs $T_{\text{H}}/T_{\text{L}}$. The exact solution of the optimum isentropic temperature ratio can be determined by Eq. (35). Moreover, the maximum exergetic efficiency that can be reached by the refrigeration cycle increases when the temperature ratio of the two reservoirs $T_{\text{H}}/T_{\text{L}}$ is raised.

The coefficient of performance (COP) of the refrigeration system versus the isentropic temperature ratio y is presented in Fig. 3. A maximum value of COP similarly occurs as the isentropic temperature ratio y varies. However, the maximum COP that is reachable is reduced when the temperature ratio of the two reservoirs $T_{\text{H}}/T_{\text{L}}$ increases. This phenomenon is opposite to that observed in Fig. 2. The reason is that the qualities of equivalent heat transfer rates at various temperatures are different. The differences can be explained by exergy concept, which states that there is a greater rate of exergy output at lower temperature given equivalent cooling rates. Therefore, given an increasing temperature ratio $T_{\text{H}}/T_{\text{L}}$, the maximum COP that is reachable decreases while the maximum exergetic efficiency η_{ex} reachable, conversely, increases. By a comparison of Figs. 2 and 3, it is evident that the exergetic efficiency provides a more appropriate description of the useful energy than the coefficient of performance does.

The relationship between the exergetic efficiency η_{ex} and coefficient of performance COP is plotted in Fig. 4. It is seen that they are linearly proportional under the same conditions. It is also noted that the maximum exergetic efficiency corresponds to the maximum coefficient of performance when

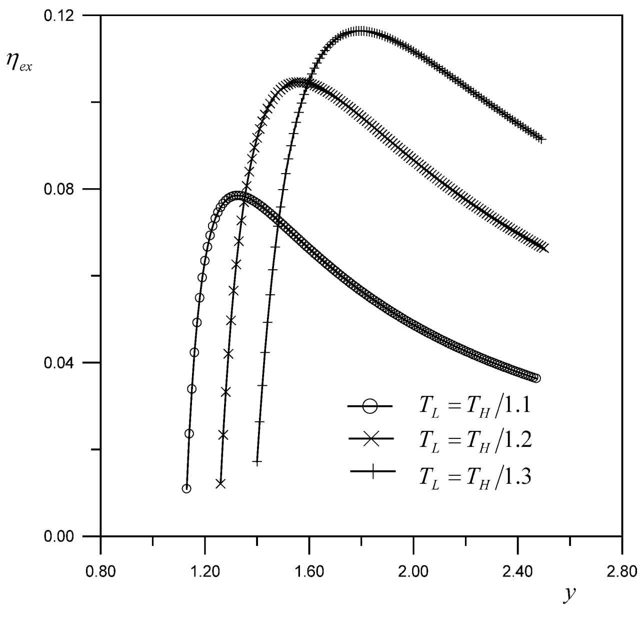

Fig. 2. The exergetic efficiency η_{ex} versus the isentropic temperature ratio y with $\eta_c = 0.85$, $\eta_e = 0.87$, $\dot{C}_I/\dot{C}_w = 0.01$, $N = 6.0$, and $N_{\text{L}}/(N_{\text{H}} + N_{\text{L}}) = 0.6$.

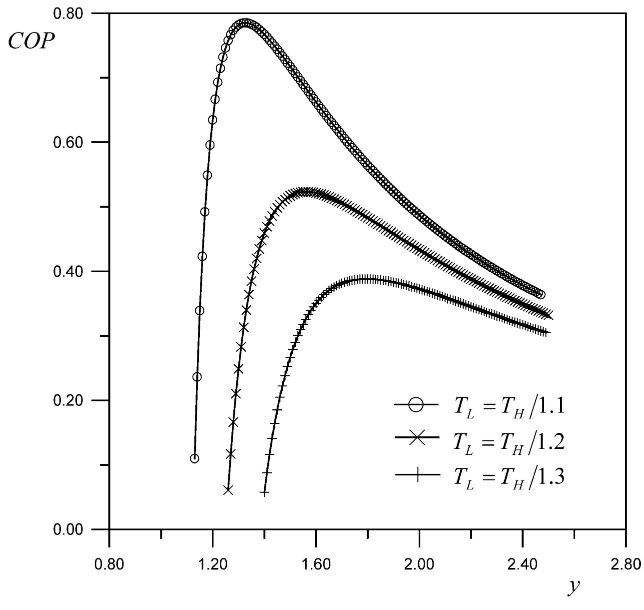

Fig. 3. Coefficient of performance COP versus the isentropic temperature ratio y with $\eta_c = 0.85$, $\eta_e = 0.87$, $\dot{C}_I/\dot{C}_w = 0.01$, $N = 6.0$, and $N_{\text{L}}/(N_{\text{H}} + N_{\text{L}}) = 0.6$.

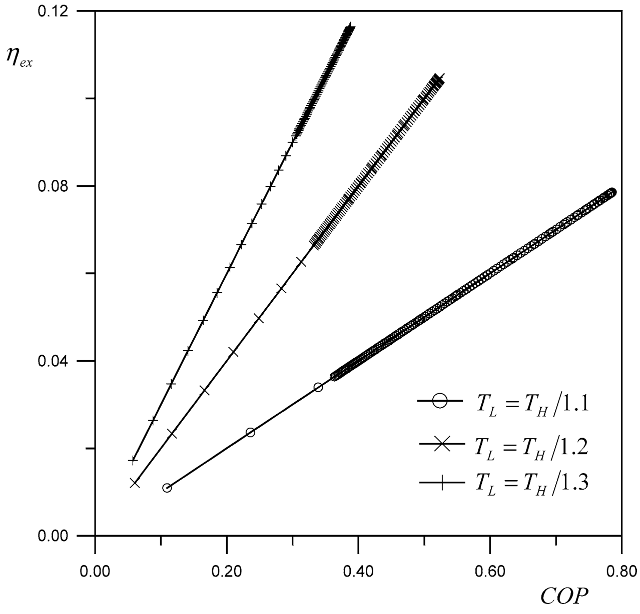

Fig. 4. The exergetic efficiency η_{ex} versus coefficient of performance *COP* with $\eta_c = 0.85$, $\eta_e = 0.87$, $\dot{C}_I/\dot{C}_w = 0.01$, $N = 6.0$, and $N_{\mathrm{L}}/(N_{\mathrm{H}} + N_{\mathrm{L}}) = 0.6$.

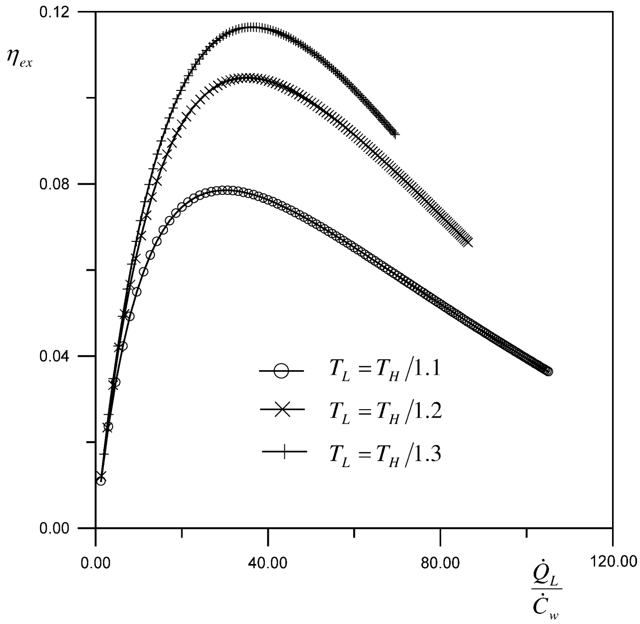

Fig. 5. The exergetic efficiency η_{ex} versus the dimensionless cooling rate $\dot{Q}_{\mathrm{L}}/\dot{C}_w$ with $\eta_c = 0.85$, $\eta_e = 0.87$, $\dot{C}_I/\dot{C}_w = 0.01$, $N = 6.0$, and $N_{\mathrm{L}}/(N_{\mathrm{H}} + N_{\mathrm{L}}) = 0.6$.

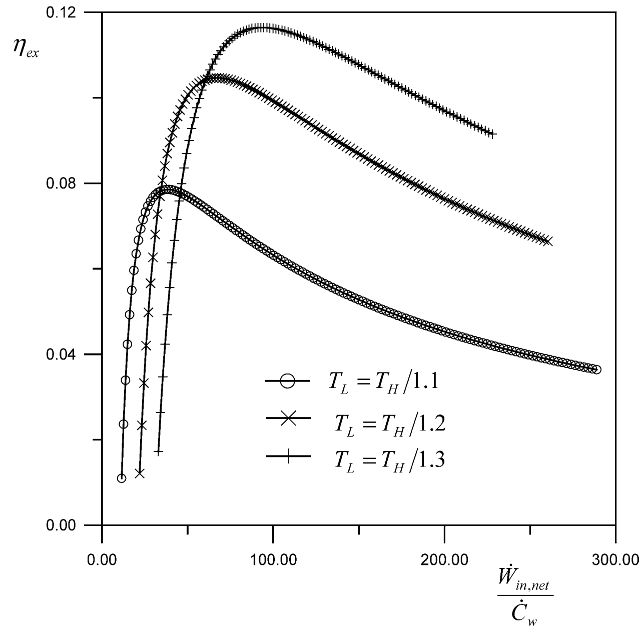

Fig. 6. The exergetic efficiency η_{ex} versus the dimensionless net power input $\dot{W}_{\mathrm{in,net}}/\dot{C}_w$ with $\eta_c = 0.85$, $\eta_e = 0.87$, $\dot{C}_I/\dot{C}_w = 0.01$, $N = 6.0$, and $N_{\mathrm{L}}/(N_{\mathrm{H}} + N_{\mathrm{L}}) = 0.6$.

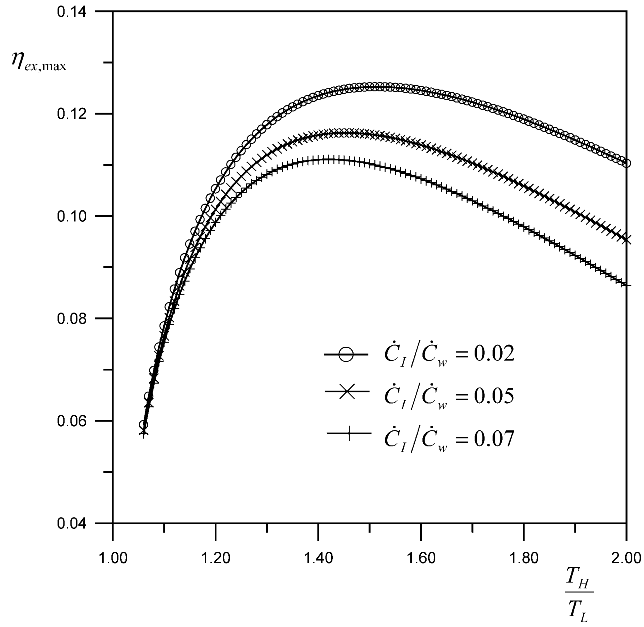

Fig. 7. The maximum exergetic efficiency $\eta_{\mathrm{ex,max}}$ versus the temperature ratio of two reservoirs $T_{\mathrm{H}}/T_{\mathrm{L}}$ with $\eta_c = 0.83$, $\eta_e = 0.87$, $N_{\mathrm{H}} = 3.2$, and $N_{\mathrm{L}} = 3.6$.

the temperature ratio of the two reservoirs is fixed. The differences represented by Figs. 2 and 3 are revealed by the various slopes of these lines.

The exergetic efficiencies plotted against the dimensionless cooling rate $\dot{Q}_{\mathrm{L}}/\dot{C}_w$ and the dimensionless net power input $\dot{W}_{\mathrm{in,net}}/\dot{C}_w$ respectively, are presented in Figs. 5 and 6. It is shown that there exists a maximum value of exergetic efficiency when either the dimensionless cooling rate or the dimensionless net power input is varied. Moreover, in order to obtain the maximum exergetic efficiency when the temperature ratio $T_{\mathrm{H}}/T_{\mathrm{L}}$ increases, the cooling rate is increased. However, in this case, the net power input must also be increased simultaneously.

The influences of the temperature ratio of two reservoirs $T_{\mathrm{H}}/T_{\mathrm{L}}$ on the maximum exergetic efficiency $\eta_{\mathrm{ex,max}}$ are shown in Fig. 7. It is seen that the maximum exergetic efficiency varies according to the temperature ratio $T_{\mathrm{H}}/T_{\mathrm{L}}$.

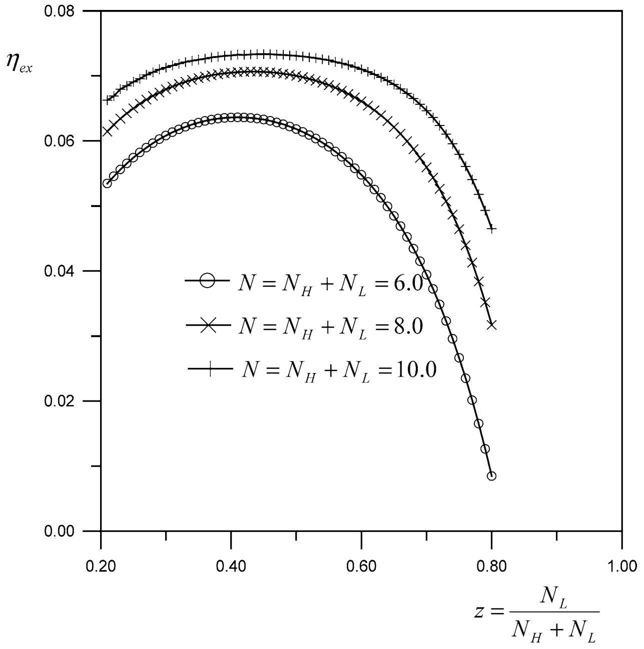

Fig. 8. The exergetic efficiency $\eta_{\rm ex}$ versus the allocation factor $z = N_{\rm L}/(N_{\rm H} + N_{\rm L})$ with $\eta_c = \eta_e = 0.87$, $\dot{C}_I/\dot{C}_w = 0.01$, $T_{\rm L} = T_{\rm H}/1.2$ and $y = 1.3$.

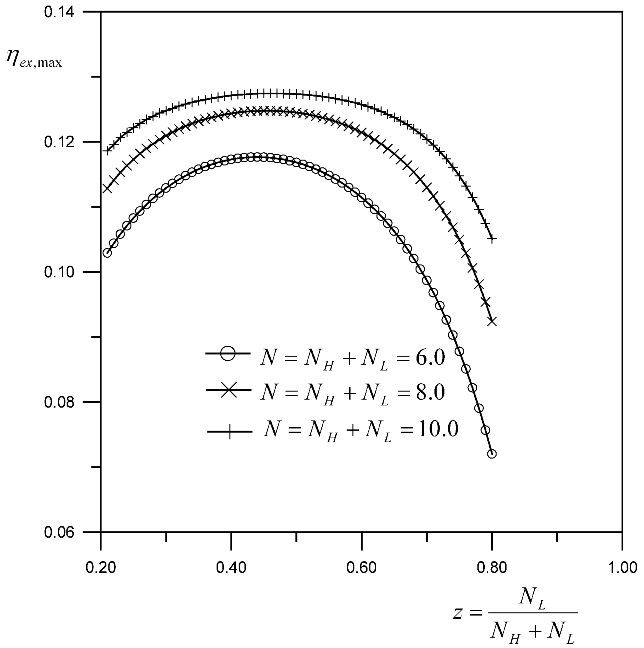

Fig. 9. The maximum exergetic efficiency $\eta_{\rm ex,max}$ versus the allocation factor $z = N_{\rm L}/(N_{\rm H} + N_{\rm L})$ with $\eta_c = \eta_e = 0.87$, $\dot{C}_I/\dot{C}_w = 0.01$, $T_{\rm L} = T_{\rm H}/1.2$ and $y = y_{\rm opt}$.

Moreover, the maximum exergetic efficiency is lower as the dimensionless heat transfer rate $\dot{C}_I/\dot{C}_w$ is a greater value. This means that in order to obtain a greater maximum exergetic efficiency, heat leak must be avoided as much as possible.

The allocation problems of a fixed total thermal conductance between two heat exchangers are calculated in Figs. 8 and 9. When the isentropic temperature ratio y is chosen to be a fixed value, there exists a maximum value of exergetic efficiency, as in Fig. 8. This phenomenon helps in allocating a fixed total thermal conductance between two heat exchangers to obtain a greater, or even maximum exergetic efficiency. Fig. 9 presents the maximum exergetic efficiency that an irreversible Brayton refrigeration cycle can reach by adjusting the allocation between two heat exchangers, where the isentropic temperature ratio is always chosen as the optimum value. Moreover, the exergetic efficiency can also be optimized by searching the optimum allocation of the heat transfer surface areas of the two heat exchangers for the fixed total heat transfer surface area of the two heat exchangers.

4. Conclusion

Exergetic efficiency optimization for a steady-flow irreversible Brayton refrigeration cycle by considering the isentropic temperature ratio and the thermal conductance allocation factor as parameters has been investigated in this paper. The maximum exergetic efficiency occurs when the isentropic temperature ratio is slightly greater than the temperature ratio of two reservoirs. The exact solution of the optimum isentropic temperature ratio can also be determined. The corresponding values of the system performances can be employed as important criteria to design and evaluate an irreversible Brayton refrigeration system. The influences of some parameters such as heat leak between heat reservoirs and temperature ratio of two reservoirs, on the maximum exergetic efficiency are discussed. Moreover, when the isentropic temperature ratio is chosen as either a fixed value or just the optimum value, the optimum allocation of a fixed total thermal conductance between two heat exchangers can be obtained by numerical calculations. The results show that the method of exergetic efficiency optimization is more practical and effective than those by traditional methods or objective indices.

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